# **A**P<sup>°</sup>

# AP<sup>®</sup> Statistics 2013 Free-Response Questions

### About the College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of more than 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT\* and the Advanced Placement Program\*. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

© 2013 The College Board. College Board, Advanced Placement Program, AP, AP Central, SAT, and the acorn logo are registered trademarks of the College Board. Admitted Class Evaluation Service and inspiring minds are trademarks owned by the College Board. All other products and services may be trademarks of their respective owners. Visit the College Board on the Web: www.collegeboard.org. Permission to use copyrighted College Board materials may be requested online at: www.collegeboard.org/inquiry/cbpermit.html.

Visit the College Board on the Web: www.collegeboard.org. AP Central is the official online home for the AP Program: apcentral.collegeboard.org.



Formulas begin on page 3. Questions begin on page 6. Tables begin on page 14.

# Formulas

(I) **Descriptive Statistics** 

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_{x} = \sqrt{\frac{1}{n-1} \sum (x_{i} - \overline{x})^{2}}$$

$$s_{p} = \sqrt{\frac{(n_{1} - 1)s_{1}^{2} + (n_{2} - 1)s_{2}^{2}}{(n_{1} - 1) + (n_{2} - 1)}}$$

$$s_p = \sqrt{(n_1 - 1) + (n_2 - 1)}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x}) (y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

 $b_0 = \overline{y} - b_1 \overline{x}$ 

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \overline{x}}{s_x} \right) \left( \frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum \left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\sum \left(x_i - \overline{x}\right)^2}}$$

#### (II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

 $E(X) = \mu_X = \sum x_i p_i$ 

$$\operatorname{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If *X* has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

 $\mu_{\chi} = np$ 

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size *n* from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

 $\mu_{\overline{x}} = \mu$ 

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$ 

Confidence interval: statistic  $\pm$  (critical value) • (standard deviation of statistic)

## Single-Sample

| Statistic         | Standard Deviation<br>of Statistic |
|-------------------|------------------------------------|
| Sample Mean       | $\frac{\sigma}{\sqrt{n}}$          |
| Sample Proportion | $\sqrt{\frac{p(1-p)}{n}}$          |

#### Two-Sample

| Statistic                        | Standard Deviation<br>of Statistic   |
|----------------------------------|--|
| Difference of sample means       | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$                                   |
|                                  | Special case when $\sigma_1 = \sigma_2$<br>$\sigma_1 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ |
| Difference of sample proportions | $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$                                   |
|                                  | Special case when $p_1 = p_2$  |
|                                  | $\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$                                       |
|                                  |  |
| Chi-square test statistic =      | $\sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$                       |

# STATISTICS SECTION II Part A Questions 1-5

#### Spend about 65 minutes on this part of the exam. Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

# Lead Levels

| 2      | 8      |
|--------|--------|
| 3      | 0      |
| 3      | 588    |
| 4      | 112    |
| 4      | 688    |
| 5<br>5 | 012234 |
|        | 99     |
| 6      | 34     |
| 6      | 68     |
|        | •      |

Key: 2|8 = 2.8 ppm

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?
- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

- 2. An administrator at a large university wants to conduct a survey to estimate the proportion of students who are satisfied with the appearance of the university buildings and grounds. The administrator is considering three methods of obtaining a sample of 500 students from the 70,000 students at the university.
  - (a) Because of financial constraints, the first method the administrator is considering consists of taking a convenience sample to keep the expenses low. A very large number of students will attend the first football game of the season, and the first 500 students who enter the football stadium could be used as a sample. Why might such a sampling method be biased in producing an estimate of the proportion of students who are satisfied with the appearance of the buildings and grounds?
  - (b) Because of the large number of students at the university, the second method the administrator is considering consists of using a computer with a random number generator to select a simple random sample of 500 students from a list of 70,000 student names. Describe how to implement such a method.
  - (c) Because stratification can often provide a more precise estimate than a simple random sample, the third method the administrator is considering consists of selecting a stratified random sample of 500 students. The university has two campuses with male and female students at each campus. Under what circumstance(s) would stratification by campus provide a more precise estimate of the proportion of students who are satisfied with the appearance of the university buildings and grounds than stratification by gender?
- 3. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs. The weights of such full cartons are approximately normally distributed with a mean of 840 grams and a standard deviation of 7.9 grams.
  - (a) What is the probability that a randomly selected full carton of Grade A eggs will weigh more than 850 grams?
  - (b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

Let the random variable *X* be the weight of a single randomly selected Grade A egg.

- i) What is the mean of *X*?
- ii) What is the standard deviation of X?

4. The Behavioral Risk Factor Surveillance System is an ongoing health survey system that tracks health conditions and risk behaviors in the United States. In one of their studies, a random sample of 8,866 adults answered the question "Do you consume five or more servings of fruits and vegetables per day?" The data are summarized by response and by age-group in the frequency table below.

| Age-Group (years) | Yes   | No    | Total |
|-------------------|-------|-------|-------|
| 18–34             | 231   | 741   | 972   |
| 35–54             | 669   | 2,242 | 2,911 |
| 55 or older       | 1,291 | 3,692 | 4,983 |
| Total             | 2,191 | 6,675 | 8,866 |

Do the data provide convincing statistical evidence that there is an association between age-group and whether or not a person consumes five or more servings of fruits and vegetables per day for adults in the United States?

© 2013 The College Board. Visit the College Board on the Web: www.collegeboard.org.

5. Psychologists interested in the relationship between meditation and health conducted a study with a random sample of 28 men who live in a large retirement community. Of the men in the sample, 11 reported that they participate in daily meditation and 17 reported that they do not participate in daily meditation.

The researchers wanted to perform a hypothesis test of

$$H_0: p_m - p_c = 0$$
  
 $H_a: p_m - p_c < 0$ ,

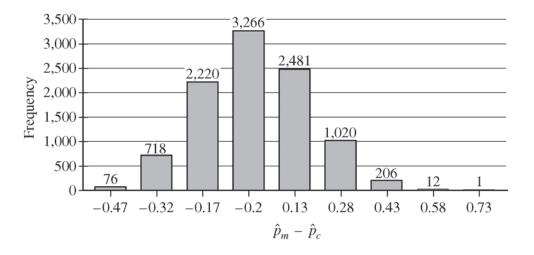
where  $p_m$  is the proportion of men with high blood pressure among all the men in the retirement community who participate in daily meditation and  $p_c$  is the proportion of men with high blood pressure among all the men in the retirement community who do not participate in daily meditation.

(a) If the study were to provide significant evidence against  $H_0$  in favor of  $H_a$ , would it be reasonable for the psychologists to conclude that daily meditation causes a reduction in blood pressure for men in the retirement community? Explain why or why not.

The psychologists found that of the 11 men in the study who participate in daily meditation, 0 had high blood pressure. Of the 17 men who do not participate in daily meditation, 8 had high blood pressure.

(b) Let  $\hat{p}_m$  represent the proportion of men with high blood pressure among those in a random sample of 11 who meditate daily, and let  $\hat{p}_c$  represent the proportion of men with high blood pressure among those in a random sample of 17 who do not meditate daily. Why is it not reasonable to use a normal approximation for the sampling distribution of  $\hat{p}_m - \hat{p}_c$ ?

Although a normal approximation cannot be used, it is possible to simulate the distribution of  $\hat{p}_m - \hat{p}_c$ . Under the assumption that the null hypothesis is true, 10,000 values of  $\hat{p}_m - \hat{p}_c$  were simulated. The histogram below shows the results of the simulation.



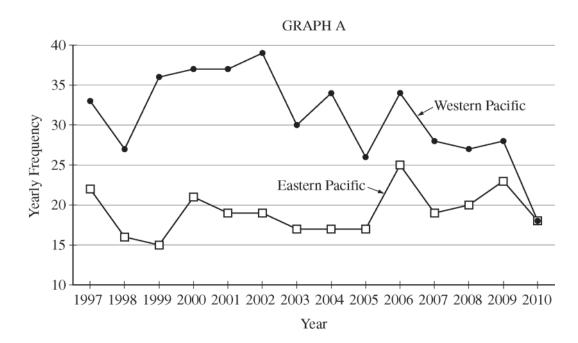
(c) Based on the results of the simulation, what can be concluded about the relationship between blood pressure and meditation among men in the retirement community?

© 2013 The College Board. Visit the College Board on the Web: www.collegeboard.org.

## STATISTICS SECTION II Part B Question 6 Spend about 25 minutes on this part of the exam. Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Tropical storms in the Pacific Ocean with sustained winds that exceed 74 miles per hour are called typhoons. Graph A below displays the number of recorded typhoons in two regions of the Pacific Ocean—the Eastern Pacific and the Western Pacific—for the years from 1997 to 2010.



- (a) Compare the distributions of yearly frequencies of typhoons for the two regions of the Pacific Ocean for the years from 1997 to 2010.
- (b) For each region, describe how the yearly frequencies changed over the time period from 1997 to 2010.

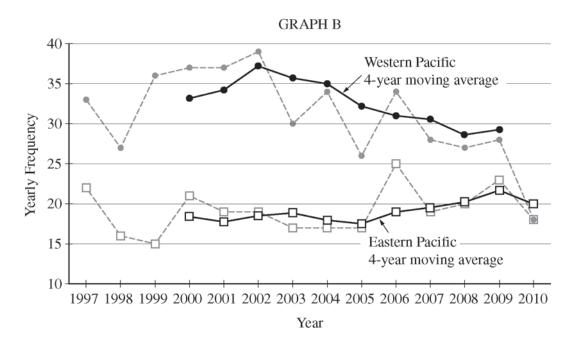
© 2013 The College Board. Visit the College Board on the Web: www.collegeboard.org.

A moving average for data collected at regular time increments is the average of data values for two or more consecutive increments. The 4-year moving averages for the typhoon data are provided in the table below. For example, the Eastern Pacific 4-year moving average for 2000 is the average of 22, 16, 15, and 21, which is equal to 18.50.

| Year | Number of<br>Typhoons in the<br>Eastern Pacific | Eastern Pacific<br>4-year moving<br>average | Number of<br>Typhoons in the<br>Western Pacific | Western Pacific<br>4-year moving<br>average |
|------|---|---|---|---|
| 1997 | 22  |   | 33  |   |
| 1998 | 16  |   | 27  |   |
| 1999 | 15  |   | 36  |   |
| 2000 | 21  | 18.50                                       | 37  | 33.25                                       |
| 2001 | 19  | 17.75                                       | 37  | 34.25                                       |
| 2002 | 19  | 18.50                                       | 39  | 37.25                                       |
| 2003 | 17  | 19.00                                       | 30  | 35.75                                       |
| 2004 | 17  | 18.00                                       | 34  | 35.00                                       |
| 2005 | 17  | 17.50                                       | 26  | 32.25                                       |
| 2006 | 25  | 19.00                                       | 34  | 31.00                                       |
| 2007 | 19  | 19.50                                       | 28  | 30.50                                       |
| 2008 | 20  | 20.25                                       | 27  | 28.75                                       |
| 2009 | 23  | 21.75                                       | 28  | 29.25                                       |
| 2010 | 18  | 20.00                                       | 18  |   |

(c) Show how to calculate the 4-year moving average for the year 2010 in the Western Pacific. Write your value in the appropriate place in the table.

(d) Graph B below shows both yearly frequencies (connected by dashed lines) and the respective 4-year moving averages (connected by solid lines). Use your answer in part (c) to complete the graph.



- (e) Consider graph B.
  - i) What information is more apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?
  - ii) What information is less apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

#### **STOP**

#### **END OF EXAM**

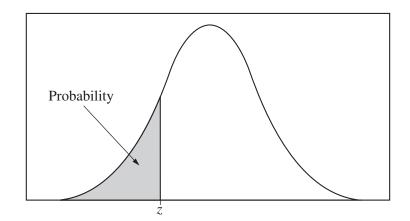


Table entry for z is the probability lying below z.

| Table A | Standard | normal | probabilities |
|---------|----------|--------|---------------|
|---------|----------|--------|---------------|

| z    | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

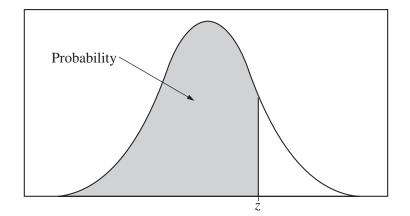


Table entry for z is the probability lying below z.

#### Table A(Continued)

|     | ``    | ,     |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |
|     | ļ     |       |       |       |       |       |       |       |       |       |

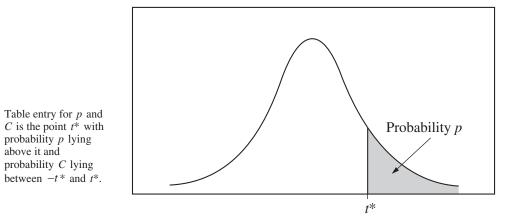


 Table B
 t distribution critical values

|           |              | Tail probability <i>p</i> |                |                |                |                |                |                |                |                |                |                |
|-----------|--------------|---------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| df        | .25          | .20                       | .15            | .10            | .05            | .025           | .02            | .01            | .005           | .0025          | .001           | .0005          |
| 1         | 1.000        | 1.376                     | 1.963          | 3.078          | 6.314          | 12.71          | 15.89          | 31.82          | 63.66          | 127.3          | 318.3          | 636.6          |
| 2         | .816         | 1.061                     | 1.386          | 1.886          | 2.920          | 4.303          | 4.849          | 6.965          | 9.925          | 14.09          | 22.33          | 31.60          |
| 3         | .765         | .978                      | 1.250          | 1.638          | 2.353          | 3.182          | 3.482          | 4.541          | 5.841          | 7.453          | 10.21          | 12.92          |
| 4         | .741         | .941                      | 1.190          | 1.533          | 2.132          | 2.776          | 2.999          | 3.747          | 4.604          | 5.598          | 7.173          | 8.610          |
| 5         | .727         | .920                      | 1.156          | 1.476          | 2.015          | 2.571          | 2.757          | 3.365          | 4.032          | 4.773          | 5.893          | 6.869          |
| 6         | .718         | .906                      | 1.134          | 1.440          | 1.943          | 2.447          | 2.612          | 3.143          | 3.707          | 4.317          | 5.208          | 5.959          |
| 7         | .711         | .896                      | 1.119          | 1.415          | 1.895          | 2.365          | 2.517          | 2.998          | 3.499          | 4.029          | 4.785          | 5.408          |
| 8         | .706         | .889                      | 1.108          | 1.397          | 1.860          | 2.306          | 2.449          | 2.896          | 3.355          | 3.833          | 4.501          | 5.041          |
| 9         | .703         | .883                      | 1.100          | 1.383          | 1.833          | 2.262          | 2.398          | 2.821          | 3.250          | 3.690          | 4.297          | 4.781          |
| 10        | .700         | .879                      | 1.093          | 1.372          | 1.812          | 2.228          | 2.359          | 2.764          | 3.169          | 3.581          | 4.144          | 4.587          |
| 11        | .697         | .876                      | 1.088          | 1.363          | 1.796          | 2.201          | 2.328          | 2.718          | 3.106          | 3.497          | 4.025          | 4.437          |
| 12        | .695         | .873                      | 1.083          | 1.356          | 1.782          | 2.179          | 2.303          | 2.681          | 3.055          | 3.428          | 3.930          | 4.318          |
| 13        | .694         | .870                      | 1.079          | 1.350          | 1.771          | 2.160          | 2.282          | 2.650          | 3.012          | 3.372          | 3.852          | 4.221          |
| 14        | .692         | .868                      | 1.076          | 1.345          | 1.761          | 2.145          | 2.264          | 2.624          | 2.977          | 3.326          | 3.787          | 4.140          |
| 15        | .691         | .866                      | 1.074          | 1.341          | 1.753          | 2.131          | 2.249          | 2.602          | 2.947          | 3.286          | 3.733          | 4.073          |
| 16        | .690         | .865                      | 1.071          | 1.337          | 1.746          | 2.120          | 2.235          | 2.583          | 2.921          | 3.252          | 3.686          | 4.015          |
| 17        | .689         | .863                      | 1.069          | 1.333          | 1.740          | 2.110          | 2.224          | 2.567          | 2.898          | 3.222          | 3.646          | 3.965          |
| 18        | .688         | .862                      | 1.067          | 1.330          | 1.734          | 2.101          | 2.214          | 2.552          | 2.878          | 3.197          | 3.611          | 3.922          |
| 19        | .688         | .861                      | 1.066          | 1.328          | 1.729          | 2.093          | 2.205          | 2.539          | 2.861          | 3.174          | 3.579          | 3.883          |
| 20        | .687         | .860                      | 1.064          | 1.325          | 1.725          | 2.086          | 2.197          | 2.528          | 2.845          | 3.153          | 3.552          | 3.850          |
| 21        | .686         | .859                      | 1.063          | 1.323          | 1.721          | 2.080          | 2.189          | 2.518          | 2.831          | 3.135          | 3.527          | 3.819          |
| 22        | .686         | .858                      | 1.061          | 1.321          | 1.717          | 2.074          | 2.183          | 2.508          | 2.819          | 3.119          | 3.505          | 3.792          |
| 23        | .685         | .858                      | 1.060          | 1.319          | 1.714          | 2.069          | 2.177          | 2.500          | 2.807          | 3.104          | 3.485          | 3.768          |
| 24        | .685         | .857                      | 1.059          | 1.318          | 1.711          | 2.064          | 2.172          | 2.492          | 2.797          | 3.091          | 3.467          | 3.745          |
| 25        | .684         | .856                      | 1.058          | 1.316          | 1.708          | 2.060          | 2.167          | 2.485          | 2.787          | 3.078          | 3.450          | 3.725          |
| 26        | .684         | .856                      | 1.058          | 1.315          | 1.706          | 2.056          | 2.162          | 2.479          | 2.779          | 3.067          | 3.435          | 3.707          |
| 27        | .684         | .855                      | 1.057          | 1.314          | 1.703          | 2.052          | 2.158          | 2.473          | 2.771          | 3.057          | 3.421          | 3.690          |
| 28        | .683         | .855                      | 1.056          | 1.313          | 1.701          | 2.048          | 2.154          | 2.467          | 2.763          | 3.047          | 3.408          | 3.674          |
| 29        | .683         | .854                      | 1.055          | 1.311          | 1.699          | 2.045          | 2.150          | 2.462          | 2.756          | 3.038          | 3.396          | 3.659          |
| 30        | .683         | .854                      | 1.055          | 1.310          | 1.697          | 2.042          | 2.147          | 2.457          | 2.750          | 3.030          | 3.385          | 3.646          |
| 40        | .681         | .851                      | 1.050          | 1.303<br>1.299 | 1.684          | 2.021          | 2.123          | 2.423<br>2.403 | 2.704<br>2.678 | 2.971          | 3.307          | 3.551          |
| 50        | .679         | .849                      | 1.047          |                | 1.676          | 2.009          | 2.109          |                |                | 2.937          | 3.261          | 3.496          |
| 60<br>80  | .679         | .848<br>.846              | 1.045<br>1.043 | 1.296<br>1.292 | 1.671<br>1.664 | 2.000<br>1.990 | 2.099<br>2.088 | 2.390<br>2.374 | 2.660<br>2.639 | 2.915<br>2.887 | 3.232<br>3.195 | 3.460<br>3.416 |
| 80<br>100 | .678<br>.677 | .846<br>.845              | 1.043          | 1.292          | 1.664          | 1.990          | 2.088          | 2.374 2.364    | 2.639          | 2.887<br>2.871 | 3.195<br>3.174 | 3.416          |
|           | .677         |                           | 1.042          | 1.290          |                | 1.984<br>1.962 | 2.081 2.056    | 2.364 2.330    | 2.626          |                | 3.174<br>3.098 | 3.390          |
| 1000<br>∞ | .675         | .842<br>.841              | 1.037          | 1.282          | 1.646<br>1.645 | 1.962          | 2.056          | 2.330          | 2.581 2.576    | 2.813<br>2.807 | 3.098<br>3.091 | 3.300<br>3.291 |
|           | 50%          | 60%                       | 70%            | 80%            | 90%            | 95%            | 96%            | 98%            | 99%            | 99.5%          | 99.8%          | 99.9%          |
|           |              |                           |                |                | (              | Confidence l   | evel C         |                |                |                |                |                |

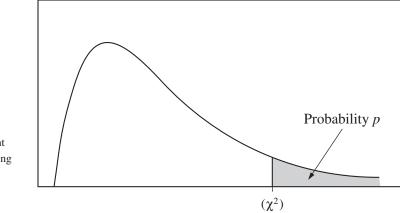


Table entry for p is the point  $(\chi^2)$  with probability p lying above it.

| Table C | $\chi^2$ | critical values |  |
|---------|----------|-----------------|--|
|---------|----------|-----------------|--|

|     |       |       |       |       |       | Tail prol | bability p |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-----------|------------|-------|-------|-------|-------|-------|
| df  | .25   | .20   | .15   | .10   | .05   | .025      | .02        | .01   | .005  | .0025 | .001  | .0005 |
| 1   | 1.32  | 1.64  | 2.07  | 2.71  | 3.84  | 5.02      | 5.41       | 6.63  | 7.88  | 9.14  | 10.83 | 12.12 |
| 2   | 2.77  | 3.22  | 3.79  | 4.61  | 5.99  | 7.38      | 7.82       | 9.21  | 10.60 | 11.98 | 13.82 | 15.20 |
| 3   | 4.11  | 4.64  | 5.32  | 6.25  | 7.81  | 9.35      | 9.84       | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4   | 5.39  | 5.99  | 6.74  | 7.78  | 9.49  | 11.14     | 11.67      | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5   | 6.63  | 7.29  | 8.12  | 9.24  | 11.07 | 12.83     | 13.39      | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6   | 7.84  | 8.56  | 9.45  | 10.64 | 12.59 | 14.45     | 15.03      | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7   | 9.04  | 9.80  | 10.75 | 12.02 | 14.07 | 16.01     | 16.62      | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8   | 10.22 | 11.03 | 12.03 | 13.36 | 15.51 | 17.53     | 18.17      | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9   | 11.39 | 12.24 | 13.29 | 14.68 | 16.92 | 19.02     | 19.68      | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10  | 12.55 | 13.44 | 14.53 | 15.99 | 18.31 | 20.48     | 21.16      | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11  | 13.70 | 14.63 | 15.77 | 17.28 | 19.68 | 21.92     | 22.62      | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12  | 14.85 | 15.81 | 16.99 | 18.55 | 21.03 | 23.34     | 24.05      | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13  | 15.98 | 16.98 | 18.20 | 19.81 | 22.36 | 24.74     | 25.47      | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14  | 17.12 | 18.15 | 19.41 | 21.06 | 23.68 | 26.12     | 26.87      | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15  | 18.25 | 19.31 | 20.60 | 22.31 | 25.00 | 27.49     | 28.26      | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16  | 19.37 | 20.47 | 21.79 | 23.54 | 26.30 | 28.85     | 29.63      | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17  | 20.49 | 21.61 | 22.98 | 24.77 | 27.59 | 30.19     | 31.00      | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18  | 21.60 | 22.76 | 24.16 | 25.99 | 28.87 | 31.53     | 32.35      | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19  | 22.72 | 23.90 | 25.33 | 27.20 | 30.14 | 32.85     | 33.69      | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20  | 23.83 | 25.04 | 26.50 | 28.41 | 31.41 | 34.17     | 35.02      | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21  | 24.93 | 26.17 | 27.66 | 29.62 | 32.67 | 35.48     | 36.34      | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22  | 26.04 | 27.30 | 28.82 | 30.81 | 33.92 | 36.78     | 37.66      | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23  | 27.14 | 28.43 | 29.98 | 32.01 | 35.17 | 38.08     | 38.97      | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24  | 28.24 | 29.55 | 31.13 | 33.20 | 36.42 | 39.36     | 40.27      | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25  | 29.34 | 30.68 | 32.28 | 34.38 | 37.65 | 40.65     | 41.57      | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26  | 30.43 | 31.79 | 33.43 | 35.56 | 38.89 | 41.92     | 42.86      | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27  | 31.53 | 32.91 | 34.57 | 36.74 | 40.11 | 43.19     | 44.14      | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28  | 32.62 | 34.03 | 35.71 | 37.92 | 41.34 | 44.46     | 45.42      | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29  | 33.71 | 35.14 | 36.85 | 39.09 | 42.56 | 45.72     | 46.69      | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30  | 34.80 | 36.25 | 37.99 | 40.26 | 43.77 | 46.98     | 47.96      | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40  | 45.62 | 47.27 | 49.24 | 51.81 | 55.76 | 59.34     | 60.44      | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50  | 56.33 | 58.16 | 60.35 | 63.17 | 67.50 | 71.42     | 72.61      | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60  | 66.98 | 68.97 | 71.34 | 74.40 | 79.08 | 83.30     | 84.58      | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80  | 88.13 | 90.41 | 93.11 | 96.58 | 101.9 | 106.6     | 108.1      | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1 | 111.7 | 114.7 | 118.5 | 124.3 | 129.6     | 131.1      | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |